BIE 5300/6300 Assignment #6
Pipe Flow Measurement

13 Oct 04 (due 18 Oct 04)

Show your calculations in an organized and neat format. Indicate any assumptions or relevant comments. E-mail the assignment to me, or drop it by my office, by 12:00 pm Monday, then I will post my solutions at 12:30 pm the same day so that you can study for the test on Tuesday.

I. You use a simple Pitot tube to measure the total head at the center of a circular pipe with an inside diameter of 336 mm. The tip of the tube points in the upstream direction. You find a total head of 42.35 m of water when connecting the Pitot tube to a manometer. Separately, you measure the pressure in the pipe at the same location, obtaining P = 413 kPa.

   (a) Estimate the velocity in the pipe at the center of the cross section.
   (b) Estimate the flow rate in the pipe, in liters per second.

II. You have a venturi connected to a manometer with mercury, whereby the manometer is connected to an upstream tap, and to a tap just at the throat of the venturi. The head differential on the mercury is 456 mm. The diameters are: D_1 = 100 mm (upstream), and D_2 = 50 mm (throat). The calibration coefficient for a "machined inlet" is C = 0.995. Calculate the flow rate through the venturi.

III. You have a sharp-crested circular orifice at a gasketed pipe flange fitting. The upstream pipe ID is D_1 = 12.0 inches and the orifice diameter is 9.05 inches. The orifice opening is centered in the pipe cross section. The upstream tap is at a distance D1 upstream of the orifice plate, and the downstream tap is at a distance ½D_1 downstream of the plate. When the taps are connected to a manometer with "blue" fluid (sg = 1.75), the head differential is observed to be 0.519 m. The water temperature is measured and found to be 8°C. Calculate the flow rate to three significant digits, taking into account the Reynold's number.

IV. You have to estimate the discharge from a partially-full horizontal pipe which discharges freely into a canal. The end of the pipe is 20 cm above the water surface in the canal. The pipe inside diameter is 35 cm, and the depth of water at the pipe end is measured, giving 13 cm. Estimate the discharge in m^3/s.

Solutions:

I. You use a simple Pitot tube to measure the total head at the center of a circular pipe with an inside diameter of 336 mm. The tip of the tube points in the upstream direction. You find a total head of 42.35 m of water when connecting the Pitot tube to a manometer. Separately, you measure the pressure in the pipe at the same location, obtaining P = 413 kPa.

(a) Velocity in the pipe at the center of the cross section.

Pipe area:

$$A = \frac{\pi(0.336)^2}{4} = 0.08867 \text{ m}^2$$

Pressure head:

$$\frac{P}{\gamma} = \frac{413 \text{ kPa}}{9.81} = 42.1 \text{ m}$$

Velocity head:

$$\frac{V^2}{2g} = 42.35 - 42.1 = 0.25 \text{ m}$$

Velocity (at center):

$$V = 2.21 \text{ m/s}$$

(b) Flow rate in the pipe, in liters per second.

The maximum flow rate would be:

$$Q_{max} = VA = (2.21)(0.08867) = 0.196 \text{ m}^3\text{/s}$$

or, 196 lps. The true flow rate is probably slightly lower than this because the velocity at the center of the cross section is greater than the average velocity, even for fully turbulent flow.

II. You have a venturi connected to a manometer with mercury, whereby the manometer is connected to an upstream tap, and to a tap just at the throat of the venturi. The head differential on the mercury is 456 mm. The diameters are: D_1 = 100 mm (upstream), and D_2 = 50 mm (throat). The calibration coefficient for a “machined inlet” is C = 0.995. Calculate the flow rate through the venturi.

First, the beta ratio is:

$$\beta = \frac{D_2}{D_1} = \frac{50}{100} = 0.50$$

The cross-sectional area is:

$$A_2 = \frac{\pi(0.05)^2}{4} = 0.001963 \text{ m}^2$$

The flow rate is:

$$Q = C_d A_2 \frac{\sqrt{2g\Delta h(sg-1)}}{\sqrt{1-\beta^4}} =$$

$$0.995(0.001963)\frac{\sqrt{2g(0.456)(13.6-1)}}{\sqrt{1-(0.50)^4}} = 0.0214 \text{ m}^3\text{/s}$$

or, 0.756 cfs.

III. You have a sharp-crested circular orifice at a gasketed pipe flange fitting. The upstream pipe ID is D_1 = 12.0 inches and the orifice diameter is 9.05 inches. The orifice opening is centered in the pipe cross section. The upstream tap is at a distance D1 upstream of the orifice plate, and the downstream tap is at a distance ½D_1 downstream of the plate. When the taps are connected to a manometer with "blue" fluid (sg = 1.75), the head differential is observed to be 0.519 m. The water temperature is measured and found to be 8°C. Calculate the flow rate to three significant digits, taking into account the Reynold's number.

First, the beta ratio is:

$$\beta = \frac{D_2}{D_1} = \frac{9.05}{12.00} = 0.7542$$

The cross-sectional area is:

$$A_2 = \frac{\pi(9.05/12)^2}{4} = 0.4467 \text{ ft}^2$$

or, 0.04150 m^2.

The kinematic viscosity is:

$$\nu = \frac{1}{83.9192(8)^2 + 20{,}707.5(8) + 551{,}173} = 1.385(10)^{-6} \text{ m}^2\text{/s}$$

Assume that the linear expansion due to pipe and element temperature is negligible (which it probably is). Next, assume a starting C_d value of 0.6.

$$Q_1 = 0.6(0.0415)\frac{\sqrt{2g(0.519)(1.75-1)}}{\sqrt{1-(0.7542)^4}} = 0.0837 \text{ m}^3\text{/s}$$

The Reynold's number is:

$$R_e = \frac{4Q}{\pi D \nu} = \frac{4(0.0837)}{\pi(0.2299)(0.000001385)} = 334{,}700$$

The first calculated C_d value is:

$$\begin{aligned} C_d &= 0.5959 + 0.0312(0.7542)^{2.1} - 0.184(0.7542)^8 \\ &\quad + \frac{0.039(0.7542)^4}{1-(0.7542)^4} - 0.0158(0.7542)^3 + \frac{91.71(0.7542)^{2.5}}{(334{,}700)^{0.75}} \\ &= 0.609 \end{aligned}$$

The updated flow rate is:

$$Q_2 = 0.609(0.0415)\frac{\sqrt{2g(0.519)(1.75-1)}}{\sqrt{1-(0.7542)^4}} = 0.0850 \text{ m}^3\text{/s}$$

The updated Reynold's number is:

$$R_e = \frac{4Q}{\pi D \nu} = \frac{4(0.0850)}{\pi(0.2299)(0.000001385)} = 339{,}900$$

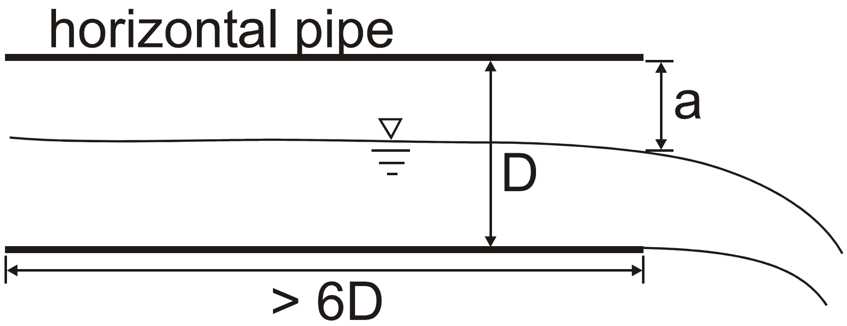
The first calculated C_d value is:

$$\begin{aligned} C_d &= 0.5959 + 0.0312(0.7542)^{2.1} - 0.184(0.7542)^8 \\ &\quad + \frac{0.039(0.7542)^4}{1-(0.7542)^4} - 0.0158(0.7542)^3 + \frac{91.71(0.7542)^{2.5}}{(339{,}900)^{0.75}} \\ &= 0.609 \end{aligned}$$

Thus, the coefficient has converged to within three significant digits after only one iteration. The flow rate is 0.0850 m^3/s (3.00 cfs).

IV. You have to estimate the discharge from a partially-full horizontal pipe which discharges freely into a canal. The end of the pipe is 20 cm above the water surface in the canal. The pipe inside diameter is 35 cm, and the depth of water at the pipe end is measured, giving 13 cm. Estimate the discharge in m^3/s.

Use the "California pipe method."



a/D = (35 - 13)/35 = 0.629 (which is greater than 0.45… OK), and

$$Q = 8.69(1-0.629)^{1.88}\left(\frac{0.35}{0.3048}\right)^{2.48} = 1.90 \text{ cfs}$$

or, 0.0538 m^3/s.